Is the Uncertainty Relation at the Root of all Mutual Exclusiveness?

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It is argued that two distinct types of complementarity are implied in Bohr's complementarity principle. While in the case of complementary variables it is the quantum mechanical uncertainty relation which is at work, the collapse hypothesis ensures this exclusiveness in the so-called wave-particle complementarity experiments. In particular it is shown that the conventional analysis of the double slit experiment which invokes the uncertainty principle to explain the absence of the simultaneous knowledge of the which-slit information and the interference pattern is incorrect and implies consequences that are quantum mechanically inconsistent.

Key words: Bohr's Complementarity Principle, Wave-Particle Duality, Uncertainty Relation, Mutual Exclusiveness, Quantum Mechanics.

Introduction

Bohr [1] formulated his complementarity principle to "harmonise the different views" on wave-particle dualism without addressing the principles of quantum mechanics (QM). Surprisingly, no reference to the actual physical principle which enforce the mutual exclusiveness (ME) of sharp knowledge of complementary pair of variables or properties was made in this pleonastic formulation. Bohr's complementarity principle (BCP), which is often superficially identified with the wave-particle dualism for historical reasons, however, is a more general concept. It is natural that the actual mechanism which enforces ME varies from one experimental situation to another.

However, in a recent letter Storey et al. [2] have concluded that the principle of complementarity is a consequence of the Heisenberg uncertainty principle. We disagree [3] with this sweeping generalisation and emphasize that two intuitive formulations of the notion of complementarity by Bohr may be classified into two distinct classes on the basis of the origin of mutual exclusiveness. These are:

Class I: In this class belong pairs of variables obtained from a Fourier transform of the state vector and any pair of non-commuting dynamical variables. These include pairs of canonically conjugate variables for which the corresponding operators satisfy the usual uncertainty relation. For other non-commuting pairs of variables uncertainty relations are also true but here the uncertainty product depends on the state vector.

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Class II: A property such as interference which depends on the superposition of a number of states, and the property associated with the "which state" information, form a pair of complementary properties. Here ME follows from the principle of collapse of wavefunction.

An important point to note is that here we refer to complementarity between a pair of *properties*, rather than a pair of *variables* as in class I. Usually it is difficult to associate specific operators corresponding to these properties [3]. The possibility of assigning suitable pair of operators was explored in [3] and we have noted that the commutation relation of such suggested pair of operators does not lead to any uncertainty type relation.

Complementary Pairs in Quantum Mechanics

Various gedanken experiments have been analysed in subsequent years, which emphasize complementarity in QM. Early examples include Heisenberg's γ-ray microscope [4], atomic de-excitation experiment [5] and Einstein's recoiling (double) slit arrangement [6]. In the first two of the examples the complementarity is between a pair of variables, and hence the origin of ME can be traced to some quantum mechanical indeterminacy relation. Heisenberg's original treatment [4] of this gedanken experiment deals with the scattering of a single photon by an individual electron. The notion of uncertainty in a single measurement is not amenable to any interpretation implied by the mathematical formalism of QM. A reformulation [7] in terms of the statistical interpretation of Heisenberg's

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thought experiment may be regarded as a meaningful illustration of the quantum mechanical indeterminacy relation and complementarity between the canonically conjugate variables x and p_x. Here, for the complementary pairs, it is impossible to have a quantum mechanical state where both the variables have sharp values.

The atomic de-excitation experiment, on the other hand, highlights a different type of uncertainty relation in QM. Even though time is not a quantum mechanical operator, the complementarity between the Fourier pair energy (E) and time (t), expressed in the time-energy uncertainty relation, may be traced to the non-commutation of the Hamiltonian and a quantum mechanical operator, say $\hat{A}$ corresponding to some dynamical variable.

From the quantum mechanical definition of the uncertainty product

$$\Delta A\,\Delta E \geq \left|\frac{1}{2i}\langle\psi\,|[\hat{A},\hat{H}]|\,\psi\rangle\right|,$$

and using the quantum mechanical equation of motion, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\hat{A}\rangle = \left|\frac{1}{i\hbar}\langle\psi\,|[\hat{A},\hat{H}]|\,\psi\rangle\right|.$$

Hence, $\Delta A\Big/\left(\frac{\mathrm{d}}{\mathrm{d}t}\langle\hat{A}\rangle\right) \geq \hbar/(2\,\Delta E)$.

Now for a particular quantum mechanical state (ψ), the r.h.s. is fixed. So, for any arbitrary $\hat{A}$, the l.h.s. has a lower bound τ, characteristic of ψ having the dimension of time. Then,

$$\tau\,\Delta E \geq \hbar/2\,.$$

Also, we have complementarity between components of angular momentum, j_x and j_y or j_y and j_z, etc. which are not canonically conjugate variables. The non-commutation relations in these cases lead to uncertainty products which depend on the state function – not the usual type of uncertainty relation. Our point is that for *complementary variables* ME follows quite clearly from the general quantum mechanical uncertainty principles.

The crux of the problem is with the class II type of pairs appearing in interferometry. Does the uncertainty relation have any role in ME in these cases? We want to emphasize here, in the first instance, that all so-called wave-particle complementarity experiments belong to class II, and QM does not guarantee the claim of complementarity between arbitrarily defined wave and particle properties [3]. It is in this sense that some authors have discussed violation of BCP [8, 9].

Wave-Particle Complementarity

The most discussed wave-particle complementarity experiment is the double slit interference experiment. Quite frequently people have used Heisenberg's uncertainty relation to explain the ME aspect. But we believe that this approach does not give a correct description.

Before examining how far these explanations are consistent with the formalism of QM, let is consider the standard arguments. In the famous example of the double slit interference experiment with electrons [10], when Feynman intends to "watch" the electron, using a scattered photon as a spy, the interference pattern gets washed out, and Feynman concludes: "If an apparatus is capable of determining which hole the electron goes through, it *cannot* be so delicate that it does not disturb the pattern in an essential way. No one has ever found (or even thought of) a way around the uncertainty principle".

The standard explanation in terms of Heisenberg's uncertainty principle contends [11] that the degree of localisation of an electron depends on the wavelength λ_p of the illuminating photon. Therefore, which-slit information is obtained when $\lambda_\mathrm{p} \leq a$ (= width of each slit). But such a light quantum will deliver to the electron a momentum which is uncertain by $\Delta p \geq \hbar/a$. So the angular dispersion of the scattered electron is: $\Delta\theta \simeq \Delta p/p \geq \hbar/(ap) = \lambda_\mathrm{el}/a$; which is obviously greater than the angular difference between the successive minima of the pattern, i.e. λ_el/d, where d is the separation between the midpoints of the two slits ($d > a$). Thus the which-slit information destroys the interference pattern.

Our first criticism is that the above type of analysis is not strictly based on the principles of QM. The uncertainty relation that follows from non-commutation of canonically conjugate variables [12] nowhere uses the concept of simultaneous approximate measurement of non-commuting observables. But this is the essence of Heisenberg's formulation [4] used in the standard explanations. In particular, what happens to a state vector after an approximate measurement of say position, is not specified by QM. Interference is basically a property of the characteristic wavefunction in a double slit experiment. Any explanation of the disappearance of the interference due to a measurement without clarifying how the statevector is being altered by the measurement, seems incomplete. In particular, Heisenberg's uncertainty relation uses a peculiar mixture of classical and quantum concepts and in most cases offers a semi-classical explanation for some quantum phenomena.

The entire analysis here envisages classical particles like passage (see the next paragraph) of the electrons (even when they do interfere!) through the double slit, while a correct quantum mechanical explanation should be obtained in terms of the quantum mechanical wavefunction (consistent with the boundary conditions on the double slit surface) of the interfering beam plus the detector states – the concept of momentum transfer being out of place.

The paths of (micro) particles in the Wilson cloud chamber, in the cathode ray oscillograph and in many other instruments can be excellently precalculated according to the laws of classical mechanics and visually observed. A moving electron in a Wilson cloud chamber actually leaves a very real cloud track. From an elementary analysis of quantum mechanical uncertainty relations one can show that an approximate track with a finite transverse extension may be ascribed to each microparticle of an ensemble. The concept of a microparticle trajectory is, therefore, only meaningful when the lateral extension can be neglected compared to the dimension of the target. If a slit of the same order of width(s) is placed across, the beam of microparticles will produce diffraction or interference pattern instead of uniform distribution. It is erroneous to think that some actual path of the microparticle does exist somewhere in a more or less narrow spatial domain and limited range of momenta, and Schrödinger (E. Schrödinger, Nobel Lecture, Dec. 12, 1933) writes: "Admittedly the individual path of a mass point loses its proper physical significance and becomes as fictitious as the individual isolated ray of light". Accordingly, quantum mechanics does not provide a framework in which we can talk about the path of a particle in a meaningful way. The path prior to the detection of any physical entity can be retrodicted but it is a matter of personal belief whether such an extrapolation of the past history can be ascribed to any physical reality or not.

Similarly in the archetypal example of a recoiling-double slit arrangement, following the Bohr-Feynman arguments [6, 10] we note that in order to observe the interference pattern, the slits must be localised with an accuracy of the order of the slit width and the corresponding uncertainty in the slit momentum is always much greater than the momentum transfer in the scattering process. Consequently the "welcher Weg" information is not available when the photons do produce an interference pattern. On this point we also recall that Bacry [8] and Hauschildt [13] have criticised the Bohr-Feynman explanation invoking Heisenberg's uncertainty principle in a similar vein.

Incidentally, we also recall Englert et al.'s [14] analysis of the double slit experiment in terms of the causal interpretation of quantum mechanics as advanced by Bohm. Some interesting points and counterpoints regarding this interpretation, in particular the "notion" of the slit through which the particle passes in the sense of causal interpretation and formal interpretation have, subsequently, been debated and discussed by Dürr et al. [15] and again by Englert et al. [16]. Although the above investigations provide some clarification about the nature of Bohm trajectories vis-a-vis the double slit experiment they, however, do not provide any clue to the understanding of the actual mechanism responsible for the mutual exclusiveness in the so-called wave-particle complementarity.

Invoking Heisenberg's uncertainty relation, alternatively, the electron beam is also considered as a classical wave with a definite wavelength $\lambda = \hbar/p$. By an approximate measurement of the position, we alter the situation only in that the wavelength of the beam is being randomly altered, and hence the interference pattern disappears as happens in the case of a classical wave. It appears that the details of Storey et al.'s [2] analysis (to be published elsewhere) can be replaced by this simpler argument. In double slit like experiments "which path"-information is obtained by a detector, which interacts with the interfering particle and there is an energy exchange, i.e. a momentum exchange. It may also be argued that such experiments basically imply a position measurement (albeit approximate, say to the extent dx) of the particle and hence induces an uncertainty in the momentum of the particle $dp_x \simeq \hbar/dx$. If $dx \ll d$, the separation between the slits, one can show that the uncertainty in momentum and hence in λ is large enough to destroy the interference. We think this picture is again not consistent with the quantum mechanical description of the phenomenon.

The mental picture emerging from the analysis in terms of Heisenberg's uncertainty relation is that each individual microparticle produces its own pattern, and if we have the which-slit information "shifted interference patterns add together, washing out the fringes" [2]. This implies a thoroughly *unacceptable conclusion that superposition is not lost even after the which-slit detection.* In actual experiment we see that each registration of the micro-entity on the screen conforms to one of the patterns (either a continuous distribution or interference fringes) depending on whether the experimental set-up is providing the which-slit information or not. Talking about the interference pattern of individual particle/entity is meaningless.

From a detail and involved analysis of three types of interferometry experiments Hauschildt [13] has concluded that the extinction of interference by a "path measurement" is a consequence of a very elementary assumption about the quantum formalism which is consistent with the projection postulate or collapse hypothesis. From a purely quantum mechanical analysis, Bohm [11] has also identified the collapse of wavefunction as the physical mechanism which enforces ME in the so-called wave-particle complementarity experiments. If both the uncertainty principle and the collapse hypothesis are accepted as alternative mechanisms enforcing ME in these experiments, *then one must be implied in the other. But either of these conclusions is wrong.*

Our second criticism is that for the disappearance of interference it is not always necessary that an exchange of energy will take place between the detector and the particle. For example, if we put a 100% efficient detector close to slit 1 and record on the screen only those events for which the detector does not click, the result does not show any interference. In this case collapse of wavefunction offers the only explanation.

For a fixed double slit with an arbitrary welcher-Weg detector, Storey et al.'s preliminary analysis at the very outset assumes the particle-detector correlated state which together with a particular "outcome of the detector measurement" [2] may well be regarded as the operational expression for the collapse hypothesis. In the remaining part of the description of arbitrary welcher-Weg detectors Storey et al. have shown, using Fourier analysis, that the maximum transferred momentum for a perfectly efficient detector cannot be less than that required by the appropriate quantum mechanical uncertainty relation. In other words, this analysis provides a sort of inner consistency of the quantum mechanical formalism. However, we note that, since the precise physical mechanism depends on the choice of the detector basis, a Fourier analysis leading to the "momentum kick amplitude distribution" [2] is not always at work. The entire analysis has some significance only for some special detectors for approximate measurements. But BCP is defined for unambiguous detection only.

The analysis of Scully et al. [17] clearly shows that the quantum mechanical formalism guarantees the validity of ME in experiments involving superposed states (class II types of complementarity). Whenever one has which-slit information, the interference pattern gets washed out. From an elaboration of this analysis, we finally conclude [3] that only the quantum mechanical formulation in terms of the wavefunction of the entire system including the detector states can present an unambiguous description of the which-slit detection scheme, and the observed complementarity in the so-called wave-particle duality experiments is enforced by the quantum mechanical collapse hypothesis.

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